Modeling Sensitivity Using Constant Eddy Viscosity and Zero Equation Turbulence models (Case Study: 60 km Length of Ibrahimia Channel, Egypt)

Mohammed Ibrahiem Ibrahiem Mohammed El-Gamal, Mohamed Ahmed Abdel Hady Eid

Abstract - In this research, a comparison between constant eddy viscosity and zero equation turbulence models using velocity and vorticity profiles was exerted. A 3D model called IRIC (International River Interface Corporative) based on an explicit finite difference method using upwind scheme was applied. Therefore, in order to calibrate and verify this model, velocities of five different cross sections of Ibrahimia channel, Egypt, were used. The hydrographic and riverbed bathymetric surveys of Ibrahimia channel were carried out by Hydraulics Research Institute “HRI” of the National Water Research Center, Ministry of Water Resources and Irrigation, Egypt, using the provided echo-sounder light boat. The velocity measurements were carried out at locations of km: 6.0, km: 26.0, km: 41.0, km: 47.0 and km: 53.0 from the upstream boundary of Ibrahimia channel. It was determined from calibration process that the final calibrated Manning's coefficient was 0.027 that gave an excellent results compared with field ones. From this research, it was found that, the error between simulated velocities using constant eddy viscosity and field data ranged about from 0.0 % to +16.67 %, but this error varied between -5.8 % and +4.76 % using zero equation turbulence model. This means that, these models give a good simulation results and could be used, and zero equation turbulence model is more accurate than constant eddy viscosity model by about 12 %.

Keywords: 3D Modeling, Eddy viscosity models, Finite difference, Turbulence models, Ibrahimia channel

1. REVIEW

Turbulence is arguably the most challenging area in fluid dynamics. The most limiting factor in channel designs is the accuracy of turbulence models for simulations of complex turbulence flows [1].

One of the most important models is called eddy viscosity model. One major drawback of the eddy viscosity subgrid-scale stress models used in large-eddy simulations is their inability to represent correctly with a single universal different turbulent field in rotating or sheared flows, near solid walls, or in transitional regimes [2].

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Prandtl in 1925 gave the concept of the mixing-length model. This model prescribed an algebraic relation for the turbulent stresses. This early development considered the cornerstone for all turbulence modeling for the next years. The mixing length model is known as an algebraic solution model or zero-equation turbulence model. To develop a more realistic mathematical model of the turbulent stresses, in 1945, Prandtl introduced the first one-equation model by proposing that the eddy viscosity depends on the turbulent kinetic energy, k, solving a differential equation to approximate the exact equation for k [3].

It is an unfortunate fact that no single turbulence model is universally accepted as being superior for all classes of flow. The choice of turbulence model depends on some considerations such as the physics encompassed in the flow, the established practice for a specific class of application, the level of accuracy required, the available computational resources and the amount of time available for the simulation. To make the most appropriate choice of model for application, the comparison between the different models and the experimental data need to understand the capabilities and limitations of the various options [4].

2. INTRODUCTION TO MODELS

The free surface flow occurring in nature is almost always turbulent. Turbulence is characterized by random fluctuating motion of the fluid masses in three dimensions. A few characteristic of the turbulence are:

- Turbulent flow is unsteady, rotational, irregular, random, dissipative, diffusive, and chaotic. The flow consists of a spectrum of different scales (eddy sizes) where largest eddies are of the order of the flow geometry (i.e. flow depth, jet width, etc); and
- Turbulent flow is always 3Dimensional.

It is important to simulate the channel using turbulence models to give a real simulation results. It is so difficult to add turbulence models to 3D model for simulating and modeling process.

- **Constant eddy viscosity model** (υ = constant) and it is not used in this study;

Joseph Boussinesq [5] introduced the concept of eddy viscosity model. He considered the first practitioner of this model (i.e. modeling the Reynolds stress). In 1887 he proposed relating the stresses of turbulence to the mean flow to deduce system of equations. Here, the Boussinesq hypothesis is applied to model the Reynolds stress term, then a new proportionality constant (turbulence eddy viscosity) $\nu_t > 0$ has been introduced [6]. These types of models called "**E**ddy **V**iscosity **M**odels (EVM's)", and could be expressed as follows:

$$-\overline{\upsilon_i'\upsilon_j'} = \upsilon_t\left(\frac{\partial\overline{\upsilon_i}}{\partial xj}+\frac{\partial\overline{\upsilon_j}}{\partial x_i}\right)-\frac{2}{3}\left(K+\upsilon_t\frac{\partial\overline{\upsilon_k}}{\partial x_k}\right)\delta_{ij}$$

which could be written in shorthand as:

$$-\overline{\upsilon_i'\upsilon_j'} = 2\upsilon_t\ S_{ij} - \frac{2}{3}\ K\ \delta_{ij} \qquad (1)$$

where:

S_{ij} = strain tensor rate (mean value);
υ_k = kinetic eddy viscosity;
υ_t = turbulence eddy viscosity;
$K = \frac{1}{2}\overline{\upsilon_i'^2}$ = turbulence kinetic energy; and
δ_{ij} = Kronecker delta.

The additional turbulence stresses in this model could be expressed by augmenting the molecular viscosity with an eddy viscosity; this could be considered a simple constant eddy viscosity model (which works well for some free shear flows such as 2D jets and mixing layers).

In eddy viscosity turbulence models the Reynolds stresses are linked to the velocity gradients via the turbulent viscosity: this relation is called the Boussinesq assumption, where the Reynolds stress tensor in the time averaged Navier-Stokes equation is replaced by the turbulent viscosity multiplied by the velocity gradients.

- **Zero equation model**, [7]

$$\upsilon_t = \frac{1}{6}u\ h \qquad (2)$$

where:
υ_t : eddy viscosity;
u : water velocity in X-direction; and
h : water depth.

3. SITE DESCRIPTION AND MODEL SET UP

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In this research Ibrahimia channel was selected. This channel has 60 km in length and with about 80 m to 90 m variable width. The hydrographic survey of selected channel was carried out by Hydraulics Research Institute “HRI” of the National Water Research Center, Ministry of water resources and Irrigation, Egypt. Using the provided echo-sounder light boat, riverbed bathymetric survey was carried out along the channel following zigzag pathway trans-sections between the two channel sides which are roughly spaced at 50 m intervals in stream wise direction. Due to the significance of the acquired measurements, differential GPS system was utilized to provide a global accuracy of nearly 1.0 m in the plan direction with a relative depth accuracy of +/-10 cm. While the applied echo-sounder system permits flow depth measurements and consequently determining bed elevation with a relative accuracy of +/- 5 cm. For shallow areas, where the flow depths are less than 0.75 m, another total station system was used which was launched on a light rubber boat (Zodiac). Then, the file of these coordinates (x, y and z coordinates) was prepared in the form of (xyz. tpo) file that would be required for the 3D simulation.

Data of five different cross sections locations at km: 6.0, km: 26.0, km: 41.0, km: 47.0 and km: 53.0 were collected, **Fig. (1.a)**. Some parts of the grid elements are illustrated in **Fig. (1.b)**. The downstream measured discharge is 413.0 m^3/sec and the downstream measured water level is 49.83 m. The simulation process was carried out for both constant eddy and zero equations models under the following conditions:

- Output time interval = 1.0 sec.;
- Calculation time step = 0.001 sec.;
- Start time of output = 0.0 sec.;
- Discharge (Q) = 413.0 m^3/sec. with downstream boundary of water level = 49.83 m;
- No periodic boundary conditions;
- Upstream velocity is calculated by uniform flow calculation;
- Water slope is calculated from geometric data; and
- Initial water surface is calculated as non-uniform flow.

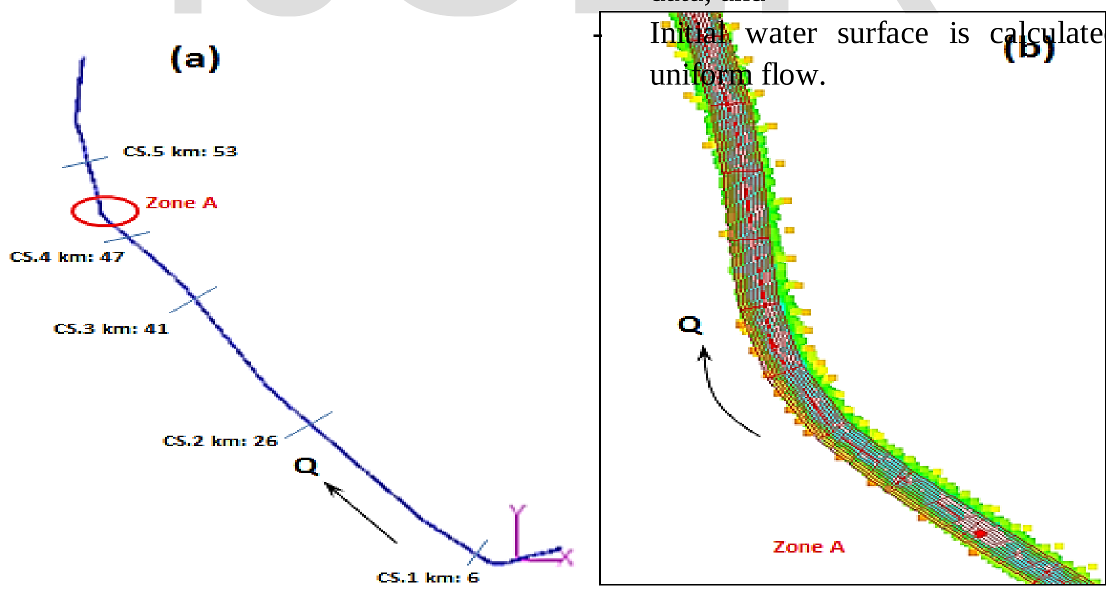

Fig. 1. (a) General layout of Ibrahimia channel and locations of five cross sections

(b) Grid elements of selected zone A.

The description of the designed grid could be given as:

- Number of streamwise nodes = 301;
- Number of cross-stream nodes in right and left floodplains = 3;
- Number of cross-stream nodes in main channel = 25;
- Number of iteration = 25; and
- Standard relaxation coefficient= 0.2.

The basic goal of mesh design is creating a representation of the water body that provides an adequate approximation of the true solution of the governing equation. The stage of network design could be finished when the contour of the whole reach can be plotted by the program. The diameter of bed material can be entered as a file with the extent of (.anc), and the standard value of critical angle of repose for bed material is used and to 0.3 [6]

- **Governing Equations**

- Momentum Equation in x-direction

$$\frac{\partial(uh)}{\partial t}+\frac{\partial(u^2h)}{\partial x}+\frac{\partial(uvh)}{\partial y}=-gh\frac{\partial H}{\partial x}-\frac{\tau_x}{\rho}+D_x$$

- Momentum Equation in y-direction

$$\frac{\partial(vh)}{\partial t}+\frac{\partial(v^2h)}{\partial y}+\frac{\partial(uvh)}{\partial x}=-gh\frac{\partial H}{\partial y}-\frac{\tau_y}{\rho}+D_y$$

- Continuity Equation

$$\frac{\partial(h)}{\partial t}+\frac{\partial(vh)}{\partial y}+\frac{\partial(uh)}{\partial x}=0.0$$

where:
v : velocity in y-direction;
u : velocity in x-direction;
τ_x : shear stress at x-direction;
τ_y : shear stress at y-direction;
t : time;
D_x : dispersion in x-direction;
D_y : dispersion in y-direction;
h : water depth at any point;
ρ : water density; and
g : gravitational acceleration.

The governing equations were converted from co-orthogonal coordinates (x and y coordinates) to represent the local stream lines into river coordinates, non-orthogonal coordinate system, (general coordinates or ξ and η coordinates). The non-orthogonal coordinate system allows more precise fitting of the coordinate system to suit arbitrary channel curvature and variable width. More importantly, the more detailed treatment of turbulence and large eddies allow predictions of time-variable behavior even for steady discharges.

4. CALIBRATION PROCESS

Van Rijn [8] defined the criteria for the selection of velocity measuring locations which are applied to this study. These criteria could be summarized as follows: located in equilibrium reach (non-degraded or aggraded); located in a straight reach of a length about five times the channel width upstream the measuring location; abundant from any secondary channel, sand bars and seasonal islands in order to avoid discharge waste through bifurcations; normal to the main flow direction; deep enough to suit with sampling and measuring apparatus; accessible and clear of natural and/or artificial obstacles; and well-defined geometrical dimensions (local depth, width, and position). According to the aforementioned conditions, the velocity data of cross section No (3) at km: 41.0 is selected for calibration process.

Cowan [9] developed a formula for estimating the effects of these factors to determine a representative Manning's value (n) for the selected channel which may be determined as follows:

$n = (n_b + n_1 + n_2 + n_3 + n_4)\, m_5$

where:

n_b = base value of n for a straight, uniform, and smooth channel in natural materials;

n_1 = factor depends on the effect of surface irregularities;

n_2 = value for variations in shape and size of the channel cross section;

n_3 = value for obstructions;

n_4 = value for vegetation and flow conditions; and

m_5 = correction factor for meandering of the channel.

From Equation (4) Manning's roughness coefficient is varied between 0.013 and 0.029. Several runs were carried out using different values of Manning's coefficient within the calculated range. The final Manning coefficient that gives excellent results compared with field data using both constant eddy viscosity and zero equation modes is 0.027. **Table (1)** gives the final results of velocities at km: 41.0 using constant eddy viscosity and zero equation turbulence models.

TABLE 1

Final calibrated velocities at km: 41.0 using both models compared with field ones.

Distance measured from right edge to left edge (m)	Field Velocities (m/sec.)	Calculated velocities using constant eddy viscosity model (m/sec.)	Calculated velocities using zero equation model (m/sec.)
7	0.56	0.58	0.56
17	0.60	0.65	0.58
27	0.66	0.68	0.66
37	0.60	0.70	0.59
47	0.63	0.70	0.66
57	0.69	0.69	0.65
67	0.60	0.63	0.59
78	0.51	0.54	0.51

Fig. (2) shows the difference between both constant eddy viscosity and zero equation turbulence models compared with field velocities for final Manning's coefficient equals 0.027 at km: 41.0.

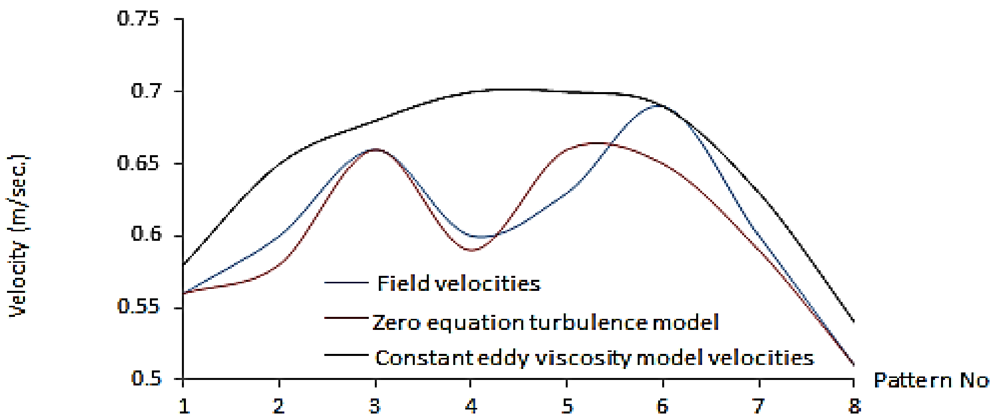

Fig. 2. Difference between both models and filed velocities at km: 41.0 for n=0.027.

From the above figure, it is found that the difference between zero equation turbulence model and the corresponding field ones ranges between -5.8 % and +4.76 %, but this range varies from 0.0 % to +16.67 % using constant eddy viscosity model. This means that, zero equation turbulence model is more accurate than constant eddy viscosity model by about 12 %.

5. MODELING PROCESS

Fig. (3) and **Fig. (4)** illustrate the difference between simulated vorticity (sec^{-1}) and velocity (m/sec.) profiles respectively at the centerline of Ibrahimia channel for both constant eddy viscosity and zero equation turbulence model.

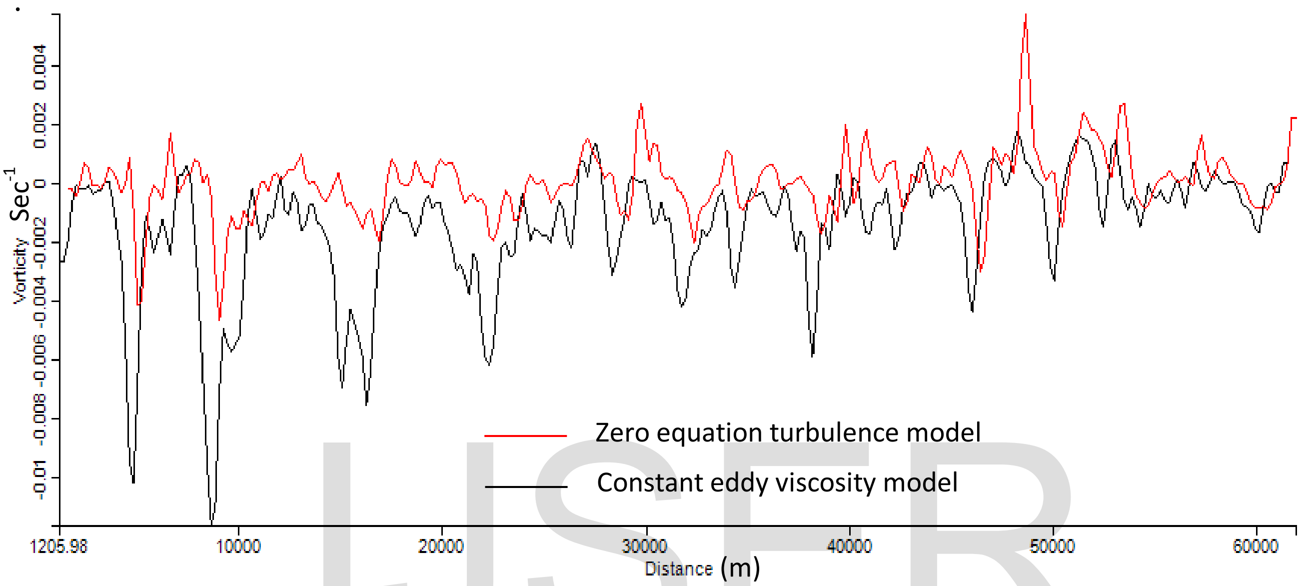

Fig. 3. Vorticity profile using constant eddy viscosity model and zero equation turbulence model at the centerline of Ibrahimia channel.

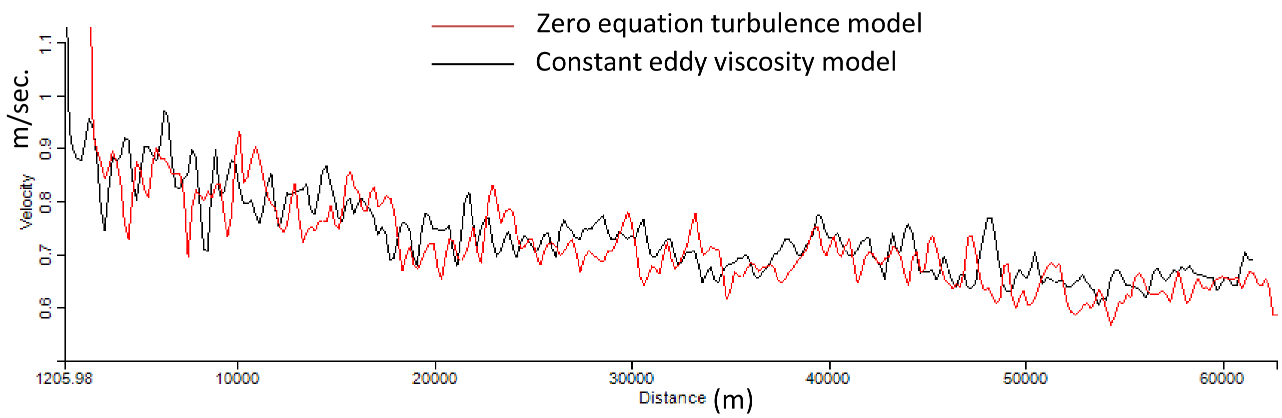

Fig. 4. Velocity profile using constant eddy viscosity model and zero equation turbulence model at the centerline of Ibrahimia channel.

Figs. (5) through (9) give the difference between computed velocities using constant eddy viscosity model and zero equation turbulence model at km: 6.0, km: 26.0, km: 41.0, km: 47.0 and km: 53.0 respectively.

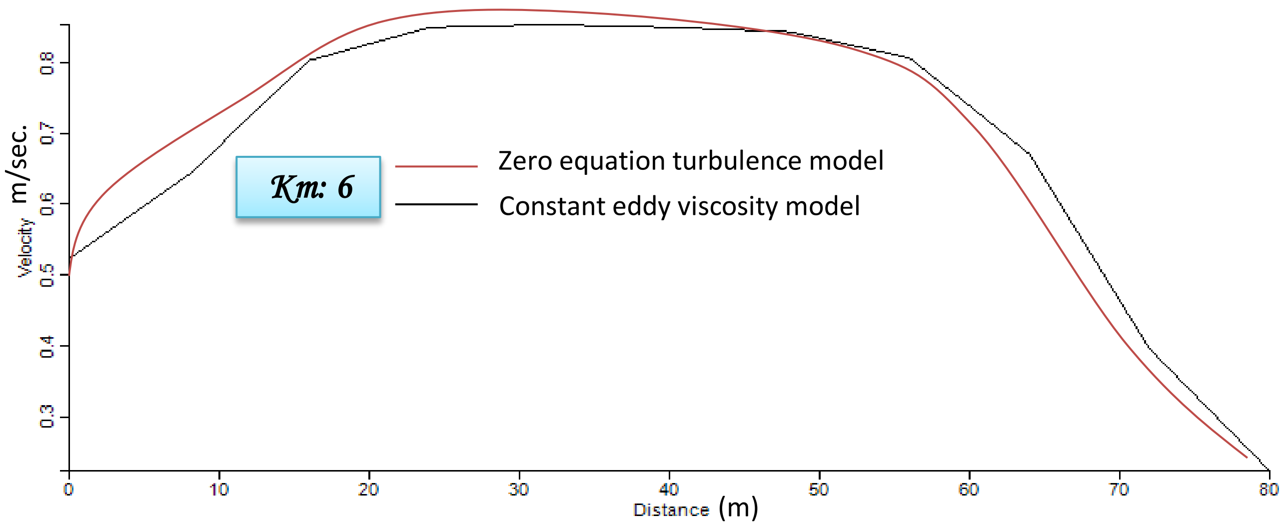

Fig. 5. Velocities using both constant eddy viscosity and zero equation models at km: 6.0.

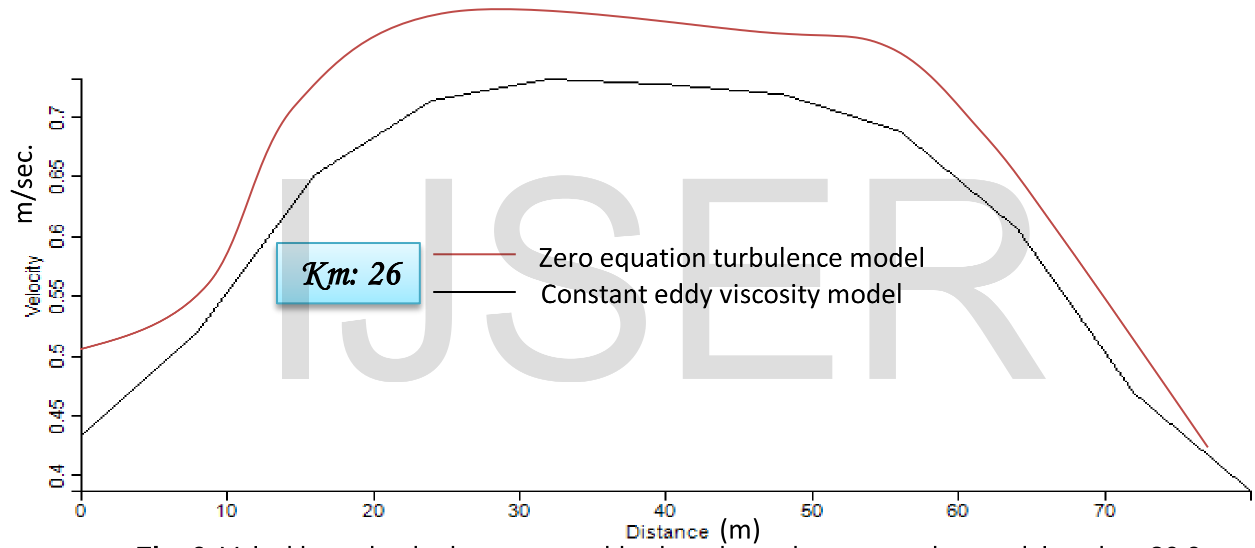

Fig. 6. Velocities using both constant eddy viscosity and zero equation models at km: 26.0.

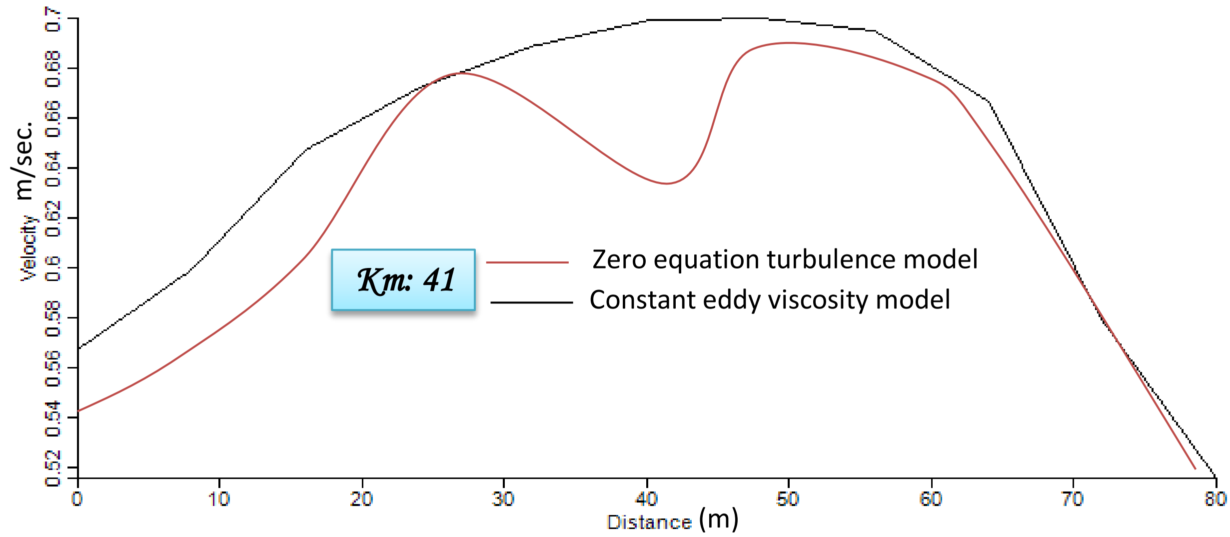

Fig. 7. Velocities using both constant eddy viscosity and zero equation models at km: 41.0.

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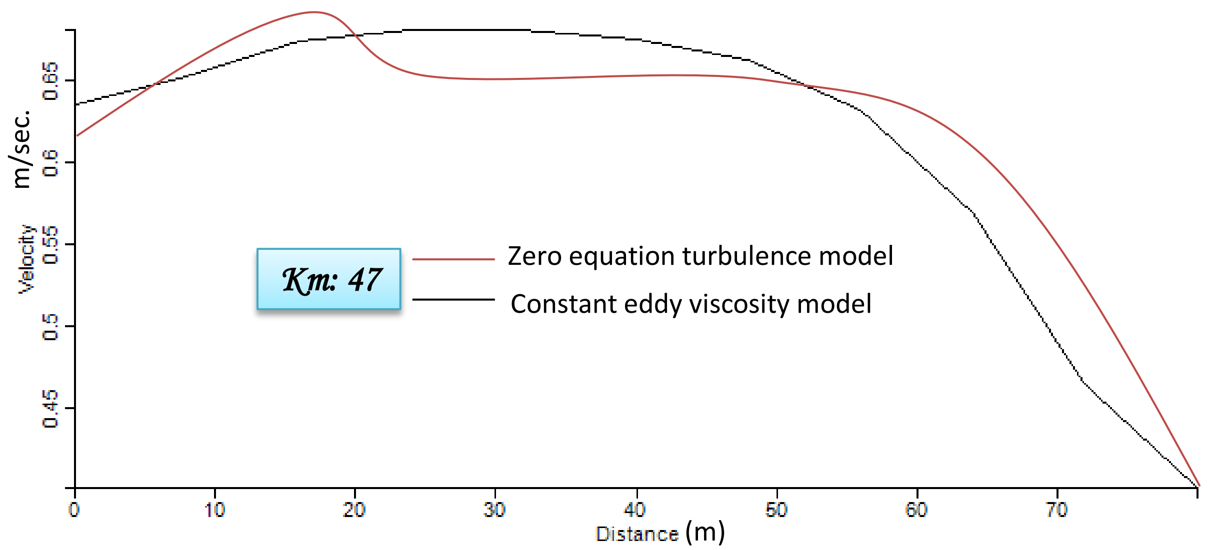

Fig. 8. Velocities using both constant eddy viscosity and zero equation models at km: 47.0.

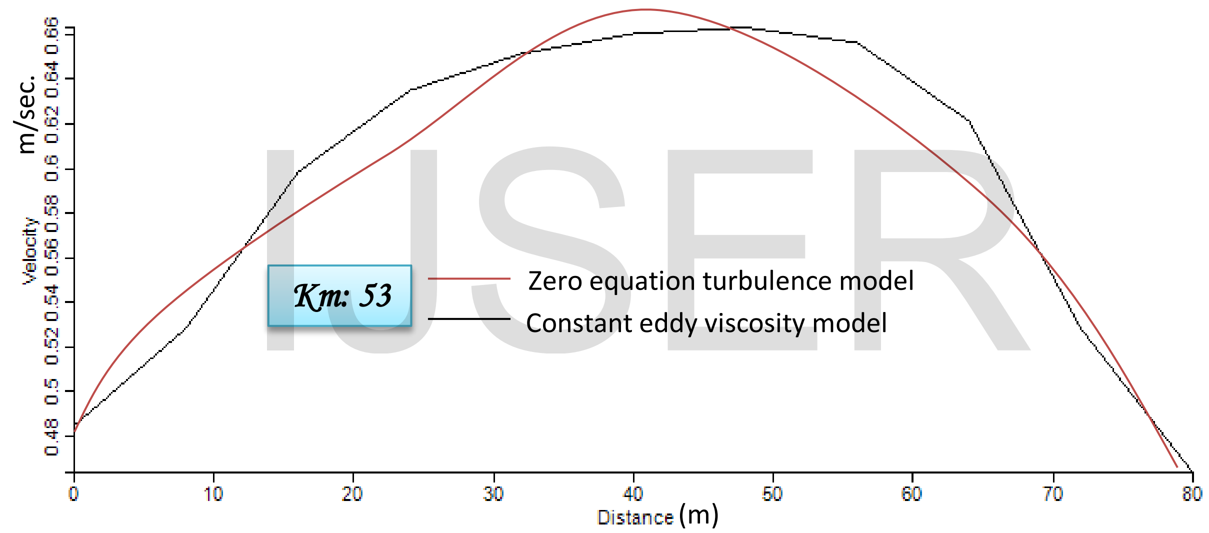

Fig. 9. Velocities using both constant eddy viscosity and zero equation models at km: 53.0.

From **Fig. (3)**, it is observed that the vorticity (sec^{-1}) value using zero equation turbulence model ranges from - 0.004 sec^{-1} to + 0.006 sec^{-1}, but this value varied between -0.02 sec^{-1} and +0.002 sec^{-1} using constant eddy viscosity model. **Figs. (4) through (9)** show that there is a noticeable difference between velocity results using constant eddy viscosity model and zero equation turbulence model. From the aforementioned analysis, it could be recommended to use zero equation turbulence model in modeling process. **Fig. (10)** exhibits the simulation of velocity, water depth and vorticity at the centerline of Ibrahimia channel using zero equation turbulence model.

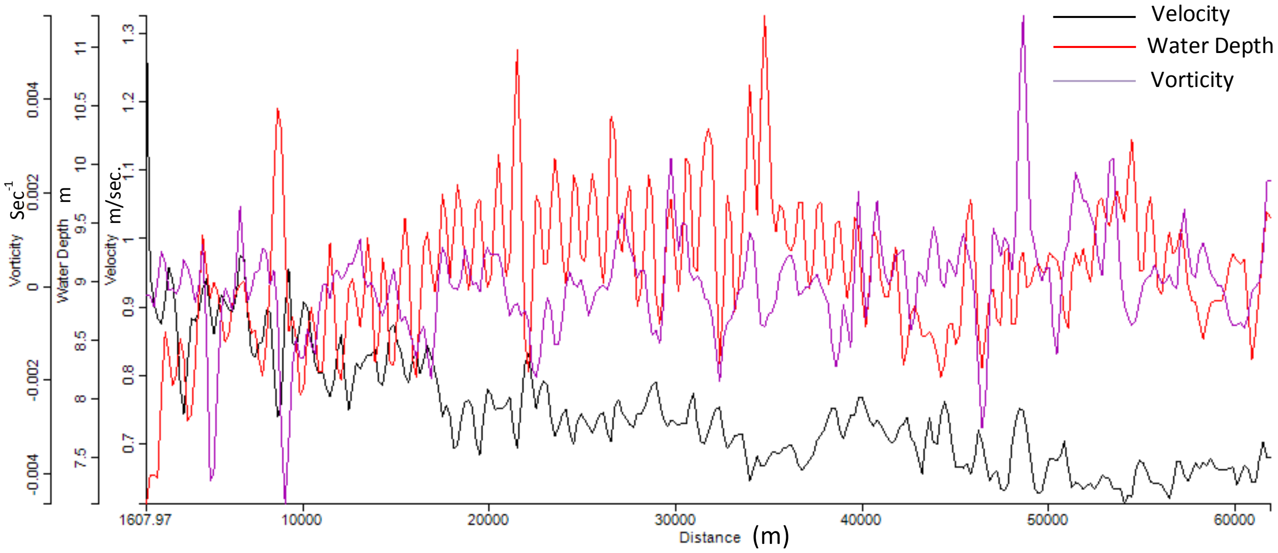

Fig. 10. Velocity, water depth and vorticity simulation at the centerline of Ibrahimia channel

From this figure, it is noticed that the fluctuation of vorticity values at the first ten kilometers is more than other values along the rest of Ibrahimia channel length. This great fluctuation affects directly the water depth and velocity.

5. CONCLUSION

In this research Ibrahimia channel with length 60 km and width ranges from 80 m to 90 m was selected as a case study to test the accuracy of two types of turbulence models (constant eddy viscosity and zero equation turbulence model). A 3D model called IRIC based on finite difference method using upwind scheme was used for the modeling process. The two models were calibrated with field velocities for a selected cross section at km: 41.0. The final calibrated Manning coefficient was 0.027 that gives the minimum error between both models and the measured velocities. Water velocities for different five cross sections at km: 6.0, km: 26.0, km: 41.0, km: 47.0 and km: 53.0 using both two models were presented. Also, to illustrate the difference between constant eddy viscosity model and zero equation turbulence model, water velocity profile at the centerline of the channel along its length (60 km) was computed. The error between zero equation turbulence model and the corresponding field ones varied between -5.8 % and +4.76 %, but this error ranged from 0.0 % to +16.67 % using constant eddy viscosity model. This means that, zero equation turbulence model is more accurate than constant eddy viscosity model by about 12 %. Finally, it could be recommended to use zero equation turbulence model in any simulation process than constant eddy viscosity model.

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